



Class 12 Maths

APPLICATION OF DERIVATIVES

Quick Revision Tables

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**Table 1: Application of Derivatives QRT – Rate of Change of Quantities**

Concept	Formula / Result
Rate of Change	If $y = f(x)$, then $\frac{dy}{dx}$ gives the rate of change of y with respect to x .
Rate at a Point	At $x = c$, the rate of change is $\left. \frac{dy}{dx} \right _{x=c} = f'(c)$
Rate with Time	If a quantity y changes with time t , its rate of change is $\frac{dy}{dt}$.
Related Rates	If x and y both depend on t , then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, provided $\frac{dx}{dt} \neq 0$.
Chain Rule in Rate Problems	If $y = f(x)$ and x varies with time, then $\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$
Sign of Rate	$\frac{dy}{dx} > 0$ means y increases as x increases; $\frac{dy}{dx} < 0$ means y decreases.
Circle	Area: $A = \pi r^2$, so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
Rectangle – Perimeter	$P = 2(x + y)$, so $\frac{dP}{dt} = 2 \left(\frac{dx}{dt} + \frac{dy}{dt} \right)$
Rectangle – Area	$A = xy$, so $\frac{dA}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$
Cube – Volume	$V = x^3$, so $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$
Cube – Surface Area	$S = 6x^2$, so $\frac{dS}{dt} = 12x \frac{dx}{dt}$
Marginal Cost	If $C(x)$ is the total cost of producing x units, then $MC = \frac{dC}{dx}$
Marginal Revenue	If $R(x)$ is the total revenue from selling x units, then $MR = \frac{dR}{dx}$

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Table 2: Application of Derivatives QRT – Increasing & Decreasing Functions

Concept	Formula / Result
Increasing Function	On an interval I , $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. As x increases, $f(x)$ also increases.
Decreasing Function	On an interval I , $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$. As x increases, $f(x)$ decreases.
Constant Function	$f(x) = k$, where k is constant, throughout the interval.
Increasing at a Point	f is increasing at x_0 if it is increasing in some open interval containing x_0 .
Decreasing at a Point	f is decreasing at x_0 if it is decreasing in some open interval containing x_0 .
Derivative Test – Increasing	If f is continuous on $[a, b]$, differentiable on (a, b) and $f'(x) > 0$ for every $x \in (a, b)$, then f is increasing on $[a, b]$.
Derivative Test – Decreasing	If f is continuous on $[a, b]$, differentiable on (a, b) and $f'(x) < 0$ for every $x \in (a, b)$, then f is decreasing on $[a, b]$.
Derivative Test – Constant	If $f'(x) = 0$ for every $x \in (a, b)$, then f is constant on $[a, b]$.
General Sign Condition	For a continuous function, $f'(x) \geq 0$ throughout an interval implies f is increasing; similarly, $f'(x) \leq 0$ implies f is decreasing.
Where to Check	To find intervals of increase/decrease, consider the points where $f'(x) = 0$ and where $f'(x)$ is not defined, and use them to divide the domain into disjoint intervals. Read more ↗



Table 3: Application of Derivatives QRT – Maxima & Minima Basics

Concept	Formula / Result
Maximum Value	A function f has a maximum value at $x = c$ if $f(x) \leq f(c)$ for all x sufficiently close to c .
Minimum Value	A function f has a minimum value at $x = c$ if $f(x) \geq f(c)$ for all x sufficiently close to c .
Local Maxima	$x = c$ is a point of local maxima (or maximum) if $f(c)$ is the greatest value of f in some open interval containing c .
Local Minima	$x = c$ is a point of local minima (or minimum) if $f(c)$ is the least value of f in some open interval containing c .
Local Maximum Value	If c is a point of local maximum, then $f(c)$ is the local maximum value of f .
Local Minimum Value	If c is a point of local minimum, then $f(c)$ is the local minimum value of f .
Turning Point	A local maximum or local minimum is a turning point where the function changes from increasing to decreasing or from decreasing to increasing.
Absolute Maximum	$f(c)$ is the absolute maximum value on a domain if $f(c) \geq f(x)$ for every x in the domain.
Absolute Minimum	$f(c)$ is the absolute minimum value on a domain if $f(c) \leq f(x)$ for every x in the domain.
Local vs Absolute	Local: compare with nearby points. Absolute: compare with every point in the given domain or interval. Read more ↗



QRT 4: Critical Points & Necessary Condition for Maxima/Minima

Concept	Formula / Result
Critical Point	A point $x = c$ in the domain of f is a critical point if $f'(c) = 0$ or f is not differentiable at c .
Stationary Point	A point $x = c$ where $f'(c) = 0$ is called a stationary point.
Necessary Condition for Local Extremum	If f has a local maximum or minimum at $x = c$, then either $f'(c) = 0$ or f is not differentiable at c .
Important Warning	$f'(c) = 0$ is not sufficient to conclude that $x = c$ is a local maximum or minimum.
Example – No Extremum	For $f(x) = x^3$, $f'(0) = 0$, but $x = 0$ is neither a local maximum nor a local minimum.
Critical Points	Critical points are points where a local maxima or minima may occur. They must be tested further using the first or second derivative test.
Derivative Does Not Exist	A point where $f'(c)$ does not exist can be a critical point and may be a local maximum or minimum.
Point of Inflexion – Key Case	If $f'(x)$ does not change sign through a critical point, the point is neither a local maximum nor a local minimum; it is called a point of inflexion .
What to Check First	Find the critical points by solving $f'(x) = 0$ and identifying points where f is not differentiable, provided the points lie in the domain.
Working Rule	Find $f'(x) \rightarrow$ find critical points $\rightarrow$ test each critical point. Do not declare a maxima or minima merely because $f'(c) = 0$. Read more ↗



Table 5: Application of Derivatives QRT – First Derivative Test

Concept	Formula / Result
First Derivative Test	Let f be continuous at a critical point $x = c$. Examine the sign of $f'(x)$ immediately to the left and right of c .
Local Maximum	If $f'(x)$ changes from positive to negative as x moves from left to right of c , then c is a point of local maxima and $f(c)$ is the local maximum value.
Local Minimum	If $f'(x)$ changes from negative to positive as x moves from left to right of c , then c is a point of local minimum and $f(c)$ is the local minimum value.
No Sign Change	If $f'(x)$ does not change sign through c , then c is neither a local maxima nor a local minima.
Sign Change: $+\rightarrow -$	Function changes from increasing to decreasing $\rightarrow$ local maxima.
Sign Change: $-\rightarrow +$	Function changes from decreasing to increasing $\rightarrow$ local minima.
No Sign Change	$+\rightarrow +$ or $-\rightarrow - \rightarrow$ neither a local maxima nor a local minima.
Point of Inflexion	If $f'(x)$ does not change sign through a critical point c , then c is a point of inflexion. For example, $f(x) = x^3$ at $x = 0$.
Sign-Chart Working	Find critical points $\rightarrow$ arrange them on the number line $\rightarrow$ choose a test point in each interval $\rightarrow$ determine the sign of $f'(x)$ $\rightarrow$ apply the sign-change rule.
Quick Sign Pattern	$+\mid c \mid - \rightarrow$ maxima. $-\mid c \mid + \rightarrow$ minima. Same signs $\rightarrow$ neither.

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Table 6: Application of Derivatives QRT – Second Derivative Test

Concept	Formula / Result
Second Derivative Test	Let $x = c$ be a critical point where $f'(c) = 0$ and $f''(x)$ exists in a neighbourhood of c . Examine $f''(c)$.
Local Maximum	If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local maximum value.
Local Minimum	If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a local minimum value.
If $f''(c) = 0$	The second derivative test is inconclusive . Use the first derivative test to determine the nature of the critical point.
Quick Sign Rule	$f''(c) < 0 \rightarrow$ local maxima. $f''(c) > 0 \rightarrow$ local minima.
What Must Be Checked	The test requires a critical point satisfying $f'(c) = 0$ and the existence of f'' in a neighbourhood of c .
First vs Second Test	First derivative test: checks the sign change of $f'(x)$. Second derivative test: uses $f''(c)$ when $f'(c) = 0$.
Working Rule	1. Find $f'(x)$. 2. Solve $f'(x) = 0$. 3. Find $f''(x)$. 4. Evaluate $f''(c)$. 5. Apply the sign rule.
Common Trap	Do not conclude “no extremum” (maxima/minima) when $f''(c) = 0$. The test simply fails to decide ; use the first derivative test.

Maths Better Tip... *Second derivative test shortcut: at a critical point with $f'(c) = 0$, $f''(c) < 0$ gives a local maximum and $f''(c) > 0$ gives a local minimum. If $f''(c) = 0$, the test is inconclusive.*



QRT 7: Absolute Maximum & Minimum on a Closed Interval

Concept	Formula / Result
Absolute Maximum	$f(c)$ is the absolute maximum value of f on $[a, b]$ if $f(c) \geq f(x)$ for every $x \in [a, b]$.
Absolute Minimum	$f(c)$ is the absolute minimum value of f on $[a, b]$ if $f(c) \leq f(x)$ for every $x \in [a, b]$.
Existence Theorem	If f is continuous on a closed interval $[a, b]$, then f has both an absolute maximum and an absolute minimum on $[a, b]$.
Where Can Absolute Extrema Occur?	For a continuous function on $[a, b]$, an absolute maximum or minimum can occur at an interior critical point or at an endpoint a or b .
Critical Points	Find points in the interval where $f'(x) = 0$ or f is not differentiable, provided the points belong to the domain.
Working Rule	1. Find all critical points of f in the interval. 2. Take the end points of the interval. 3. Evaluate f at all these points. 4. The greatest value is the absolute maximum; the least value is the absolute minimum.
Candidate Values	Compare $f(a)$, $f(b)$ and $f(c)$ for every critical point c in the interval.
Point vs Value	The point is the value of x where the extremum occurs; the corresponding value is $f(c)$.
Repeated Values	If the greatest or least value occurs at more than one point, all such points give the corresponding absolute extremum value.

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QRT 8: Applications of Maxima & Minima

Concept	Formula / Result
Optimization	Maxima/minima are used to find the largest or smallest possible value of a quantity such as distance, area, volume, cost or profit, subject to given conditions.
Objective Function	Express the quantity to be maximised or minimised as a function $y = f(x)$ of one variable.
Use the Constraint	Use the given condition to eliminate other variables and express the required quantity in terms of a single variable.
Domain	Determine the physically meaningful values of the variable from the conditions of the problem before finding the extremum.
Critical Point	Find $f'(x)$ and solve $f'(x) = 0$. Also check points where f is not differentiable, if they lie in the relevant domain.
Test for Maximum / Minimum	Use the first derivative test or second derivative test to determine the nature of the critical point.
Distance Problems	Write the distance or its square as a function of one variable. Minimising d^2 also minimises the positive distance d .
Area Problems	Express the area in terms of one variable using the given geometric constraint, then differentiate to find its maximum or minimum.
Volume Problems	Express the volume in terms of one variable using the given constraint, then find the critical point and test it.
Surface Area Problems	Express the required surface area as a function of one variable using the given dimensions or constraint, then optimise it.

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QRT 9: Important Mensuration & Optimisation Results

Figure / Problem	Formula / Important Result
Circle	Area $A = \pi r^2$; circumference $C = 2\pi r$
Sphere	Volume $V = \frac{4}{3}\pi r^3$; surface area $S = 4\pi r^2$
Cylinder	Volume $V = \pi r^2 h$; curved surface area $2\pi r h$; total surface area $2\pi r(r + h)$
Cone	Volume $V = \frac{1}{3}\pi r^2 h$; curved surface area $\pi r l$; $l^2 = r^2 + h^2$
Rectangle	Area $A = xy$; perimeter $P = 2(x + y)$
Trapezium	Area $A = \frac{1}{2}(a + b)h$, where a, b are the parallel sides.
Open Box	After cutting squares of side x from a rectangular sheet $l \times b$, $V = x(l - 2x)(b - 2x)$
Distance	To minimise a positive distance d , it is often simpler to minimise d^2 .
Fixed Sum – Maximum Product	For two positive numbers with a fixed sum, their product is maximum when the two numbers are equal.
Rectangle in a Circle	Among rectangles inscribed in a given fixed circle, the square has maximum area.
Cylinder in a Sphere	For the cylinder of maximum volume inscribed in a sphere of radius R , its height is $\frac{2R}{\sqrt{3}}$.
Cylinder in a Cone	For the cylinder of maximum volume inscribed in a right circular cone, the height of the cylinder is one-third of the height of the cone.

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Table 10: Application of Derivatives QRT – Quick Problem-Solving Results & Common Traps

Quick Check	Result / What to Do
Find Increasing Intervals	Find $f'(x) \rightarrow$ find points where $f'(x) = 0$ or f is not differentiable $\rightarrow$ divide the domain into intervals $\rightarrow$ check the sign of $f'(x)$.
Increasing / Decreasing	$f'(x) > 0 \rightarrow$ increasing; $f'(x) < 0 \rightarrow$ decreasing.
Critical Point	Check points where $f'(x) = 0$ or f is not differentiable, provided they belong to the domain.
Critical Point $\neq$ Extremum	Finding $f'(c) = 0$ only gives a candidate. The point must be tested.
First Derivative Test	$+\rightarrow - \rightarrow$ local maximum; $-\rightarrow + \rightarrow$ local minimum; no sign change $\rightarrow$ neither.
Second Derivative Test	At $f'(c) = 0$: $f''(c) < 0 \rightarrow$ maximum; $f''(c) > 0 \rightarrow$ minimum; $f''(c) = 0 \rightarrow$ inconclusive.
Absolute Maximum / Minimum	On a closed interval, evaluate f at all critical points and both endpoints , then compare the values.
Local vs Absolute	Local extremum $\rightarrow$ compare nearby values. Absolute extremum $\rightarrow$ compare values over the entire given domain or interval.
Optimisation Workflow	Objective $\rightarrow$ constraint $\rightarrow$ one-variable function $\rightarrow$ domain $\rightarrow$ critical points $\rightarrow$ test $\rightarrow$ required value.
Related Rates	Write the relation $\rightarrow$ differentiate with respect to $t \rightarrow$ substitute the given values/rates $\rightarrow$ state the answer with units.

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Quick Questions – Test Yourself

What does $\frac{dy}{dx}$ represent if y is a function of x ?

+

If the radius of a circle changes with time, how do you find the rate of change of its area?

+

If $f'(x) > 0$ throughout an interval, what can be concluded about f ?

+

If $f'(x) < 0$ throughout an interval, what can be concluded?

+

What is a critical point of a function?

+

Does $f'(c) = 0$ always mean that $x = c$ is a maximum or minimum?

+

What sign change of $f'(x)$ indicates a local maximum?

+

What sign change of $f'(x)$ indicates a local minimum?

+

What if $f'(x)$ has the same sign on both sides of a critical point?

+

If $f'(c) = 0$ and $f''(c) < 0$, what is the nature of $f(c)$?

+

If $f'(c) = 0$ and $f''(c) > 0$, what is the nature of $f(c)$?

+

What does $f''(c) = 0$ tell us in the second derivative test?

+



Quick Questions – Test Yourself

Maths Better Tip... Try to answer each question before opening it. Then click the arrow to reveal the answer and check yourself.

How do you find the absolute maximum and minimum of a continuous function on $[a, b]$? +

Why must the endpoints be checked when finding absolute extrema on a closed interval? +

What is the basic strategy for solving an optimisation problem? +

Why is it often easier to minimise d^2 instead of distance d ? +

If $C(x)$ is cost and $R(x)$ is revenue, what is the profit function? +

What is the marginal cost if $C(x)$ is the total cost of producing x units? +

If two quantities x and y both depend on time t , how can $\frac{dy}{dx}$ be found? +

What is the difference between a local maximum and an absolute maximum? +

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