Class 12 Maths

RELATIONS AND FUNCTIONS

Quick Revision Tables

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Table 1: Relations and Functions QRT – Basics

Concept	Formula / Result
Relation	A relation **R from A to B** is an arbitrary subset of the Cartesian product $A \times B$.
Cartesian Product	The Cartesian product $A \times B$ consists of ordered pairs (a, b), where $a \in A$ and $b \in B$.
Relation from A to B	If $R \subseteq A \times B$, then R is a relation from A to B.
Related Elements	If $(a, b) \in R$, then a is said to be related to b under R, written as aRb.
Relation in a Set	A relation **in a set A** is a subset of $A \times A$.
Empty Relation	A relation R in A is an **empty relation** if no element of A is related to any element of A, i.e. $R = \varnothing \subset A \times A$.
Universal Relation	A relation R in A is a **universal relation** if every element of A is related to every element of A, i.e. $R = A \times A$.
Trivial Relations	The **empty relation** and the **universal relation** are sometimes called trivial relations.

Maths Better Tip... *Remember: a relation from A to B is simply a subset of $A \times B$. So every relation is a collection of ordered pairs.*

QRT 2: Types of Relations

Type	Condition / Result
Reflexive Relation	A relation **R in a set A** is reflexive if $(a, a) \in R$ for every $a \in A$.
Symmetric Relation	A relation R in A is symmetric if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R$ for all $a_1, a_2 \in A$.
Transitive Relation	A relation R in A is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.
Checking Reflexivity	For every $a \in A$, check whether the ordered pair (a, a) belongs to R. If any such pair is missing, R is **not reflexive**.
Checking Symmetry	Whenever $(a, b) \in R$, the reverse pair (b, a) must also belong to R. Otherwise, R is **not symmetric**.
Checking Transitivity	Whenever $(a, b) \in R$ and $(b, c) \in R$, the pair (a, c) must also belong to R. Otherwise, R is **not transitive**.
Three Properties	A relation may possess **one, two, all three, or none** of the properties: reflexive, symmetric and transitive.

Maths Better Tip... *For quick checking: reflexive means "self", symmetric means "reverse", and transitive means "chain". These three simple ideas make it easier to remember the formal conditions.*

Table 3: Relations and Functions QRT – Equivalence Relations

Concept	Condition / Result
Equivalence Relation	A relation R in a set A is an **equivalence relation** if it is **reflexive, symmetric and transitive**.
Equivalence Relation Test	To show that R is an equivalence relation, verify all three properties: **reflexivity, symmetry and transitivity**.
Equivalence Class	If R is an equivalence relation in A, the **equivalence class** of $a \in A$ is $[a] = \{x \in A : (a, x) \in R\}$.
Elements of an Equivalence Class	The equivalence class $[a]$ consists of all elements of A that are related to a under R.
Class of a Related Element	If aRb, then $[a] = [b]$.
Equivalent Elements	Two elements $a, b \in A$ belong to the same equivalence class precisely when aRb.
Distinct Equivalence Classes	Two equivalence classes are either **identical or disjoint**; they cannot have some common elements and some different elements.
Equivalence Classes	The equivalence relation divides the set A into mutually disjoint equivalence classes.

Maths Better Tip... *Remember: an equivalence relation must satisfy all three properties — reflexive, symmetric and transitive. It divides the set into equivalence classes, and any two such classes are either identical or disjoint.*

Example: Consider the relation $aRb \iff a - b$ is divisible by 2 on $A = \{1, 2, 3, 4\}$. Since $1R3$, $[1] = [3] = \{1, 3\}$. On the other hand, $1 \not R 2$, and $[1] = \{1, 3\}, \quad [2] = \{2, 4\}$, so the two equivalence classes are different and disjoint.

Table 4: Relations and Functions QRT – Types of Functions

Type	Condition / Result
One-One (Injective)	A function $f : X \to Y$ is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$. Thus, distinct elements of X have distinct images.
Many-One	A function is many-one if two or more distinct elements of the domain can have the same image, i.e. $x_1 \neq x_2$ but $f(x_1) = f(x_2)$ for some $x_1, x_2 \in X$.
Onto (Surjective)	A function $f : X \to Y$ is onto if every element of Y is the image of some element of X. Equivalently, $\mathrm{Range}(f) = Y$.
Into	A function $f : X \to Y$ is into if at least one element of the co-domain is not an image of any element of the domain, i.e. $\mathrm{Range}(f) \subsetneq Y$.
Bijective	A function is **bijective** if it is both **one-one and onto**.
One-One and Onto	For a bijective function, distinct elements of the domain have distinct images and **every element of the co-domain is an image** of some element of the domain.
Finite Set Result	For a finite set X, a function $f : X \to X$ is one-one **if and only if** it is onto. This result need not hold for infinite sets.

Note: *One-one concerns whether two domain elements can have the same image; onto concerns whether every co-domain element is an image. So always check both conditions separately.*

Table 5: Relations and Functions QRT – Composition of Functions

Concept	Formula / Result
Composition of Functions	If $f : A \to B$ and $g : B \to C$, then the composition of f and g, denoted by $g \circ f$, is a function from A to C.
Composition Formula	$(g \circ f)(x) = g(f(x)), \quad \forall x \in A.$
Order of Composition	In $g \circ f$, apply f **first**, followed by g.
Domain of $g \circ f$	The composition $g \circ f$ is defined when the output of f lies in the domain of g.
Codomain of $g \circ f$	If $f : A \to B$ and $g : B \to C$, then $g \circ f : A \to C$.
Reverse Composition	Similarly, $(f \circ g)(x) = f(g(x))$, whenever the composition is defined.
Composition Is Not Commutative	In general, $g \circ f \neq f \circ g$, even when both compositions are defined.
Identity Function	The identity function on a set A, denoted by I_A, is defined by $I_A(x) = x$ for every $x \in A$.
Composition with Identity	For $f : A \to B$, $f \circ I_A = f$ and $I_B \circ f = f$.

Maths Better Tip... *Remember the order: in $g \circ f$, **f acts first and then g.** Also, never assume $g \circ f = f \circ g$; composition of functions is not commutative in general.*

Table 6: Invertible Functions & Inverse

Concept	Formula / Result
Invertible Function	A function $f : A \to B$ is **invertible** if there exists a function $g : B \to A$ such that $g \circ f = I_A$ and $f \circ g = I_B$.
Inverse Function	If $f : A \to B$ is invertible, its inverse is denoted by $f^{-1} : B \to A$ and satisfies $f^{-1} \circ f = I_A$ and $f \circ f^{-1} = I_B$.
Condition for Invertibility	A function is invertible **if and only if** it is both **one-one and onto**, i.e. bijective.
Inverse of an Inverse	If f is invertible, then $(f^{-1})^{-1} = f$.
Inverse of a Composition	If f and g are invertible functions, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
Inverse of Identity	The identity function is its own inverse: $I_A^{-1} = I_A$.
Inverse Function Values	If $f(a) = b$, then $f^{-1}(b) = a$.

Maths Better Tip... *Remember: a function has an inverse only when it is* ***bijective*** *— both one-one and onto. Also, the inverse reverses the direction: $f : A \to B$ gives $f^{-1} : B \to A$.*

Table 7: Relations and Functions QRT – Number of Relations

Concept	Formula / Result
Basic Setup	Let $n(A) = m$ and $n(B) = n$.
Cartesian Product	$n(A \times B) = mn$.
Number of Relations	The number of relations from A to B is 2^{mn}, since every relation is a subset of $A \times B$.
Reflexive Relations	The number of reflexive relations on a set A with m elements is 2^{m^2-m}.
Symmetric Relations	The number of symmetric relations on a set A with m elements is $2^{\frac{m(m+1)}{2}}$.
Transitive Relations	There is **no simple general formula** for the number of transitive relations on a set with m elements.
Equivalence Relations	The number of equivalence relations on a set with m elements is the **Bell number** B_m.
Bell Numbers	$B_1 = 1, \quad B_2 = 2, \quad B_3 = 5, \quad B_4 = 15.$

Note: *These counting results apply to finite sets and go beyond the core NCERT syllabus. They are included because they are useful for MCQs, CUET and other competitive-exam questions.*

Table 8: Relations and Functions QRT – Number of Functions

Concept	Formula / Result
Basic Setup	Let $n(A) = m$ and $n(B) = n$.
Total Functions	The number of functions from A to B is n^m.
One-One Functions	If $m \leq n$, the number of one-one functions from A to B is ${}^nP_m = \dfrac{n!}{(n-m)!}$.
One-One: Impossible	If $m > n$, the number of one-one functions is 0, since the domain has more elements than the co-domain.
Many-One Functions	Number of many-one functions = $\text{Total functions} - \text{one-one functions}$.
Onto: Equal Size	If $m = n$, the number of onto functions is $n!$.
Onto: Impossible	If $m < n$, the number of onto functions is 0, since the domain has fewer elements than the co-domain.
Onto: General Case	If $m > n$, the number of onto functions is $\sum_{k=0}^{n} (-1)^k \, {}^nC_k \, (n-k)^m$, by the **Inclusion–Exclusion Principle**.
Into Functions	Number of into functions = $\text{Total functions} - \text{onto functions}$.
Bijective Functions	Bijective functions are possible only when $m = n$; their number is $n!$.

Maths Better Tip... *For counting functions, remember the basic order:* ***total → one-one → onto → into → bijective****. First compare the sizes of the domain and co-domain to see which types are possible.*

Table 9: Relations and Functions QRT – Important Results & Quick Recall

Concept	Result / Quick Recall
Intersection of Equivalence Relations	If R_1 and R_2 are equivalence relations in a set A, then $R_1 \cap R_2$ is also an **equivalence relation**.
Relation Induced by a Function	For a function $f : X \to Y$, the relation defined by $aRb \iff f(a) = f(b)$ is an **equivalence relation** on X.
One-One Function on a Finite Set	A one-one function from a finite set A to itself is a **permutation** of the elements of A.
Equality of Functions	Two functions $f : A \to B$ and $g : A \to B$ are equal if $f(a) = g(a)$ for every $a \in A$.
Sum of One-One Functions	The sum of two one-one functions need **not** be one-one.
Sum of Onto Functions	The sum of two onto functions need **not** be onto.
Composition and Invertibility	If $f : A \to B$ has an inverse $g : B \to A$, then $g \circ f = I_A$ and $f \circ g = I_B$.
Invertibility Test	A function is invertible **if and only if** it is both one-one and onto.

Maths Better Tip... *Do not assume that familiar properties are preserved under function operations. In particular, the sum of two one-one functions need not be one-one, and the sum of two onto functions need not be onto.*

Quick Questions — Test Yourself

Maths Better Tip... *Try to answer each question before opening it. Then click the arrow to reveal the answer and check yourself.*

If $n(A) = m$ and $n(B) = n$, how many relations are possible from A to B? +

What three properties must a relation satisfy to be an equivalence relation? +

If aRb for an equivalence relation R, what can you say about their equivalence classes? +

If $f : A \to B$ is one-one, can two distinct elements of A have the same image? +

In $g \circ f$, which function acts first? +

Is $g \circ f$ always equal to $f \circ g$? +

When is a function $f : A \to B$ invertible? +

If $f(a) = b$, what is $f^{-1}(b)$ when f is invertible? +

If $n(A) = m$ and $n(B) = n$, how many one-one functions can be defined from A to B when $m \leq n$? +

If $m = n$, how many bijective functions are there from A to B? +

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